Analyzing an Agent-Based Model for House-Hunting in Ant Colonies

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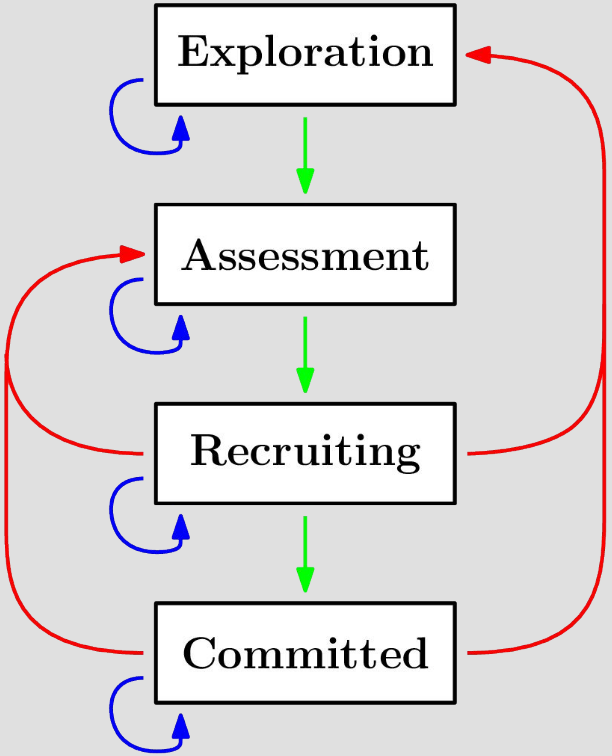
The House-Hunting Algorithm

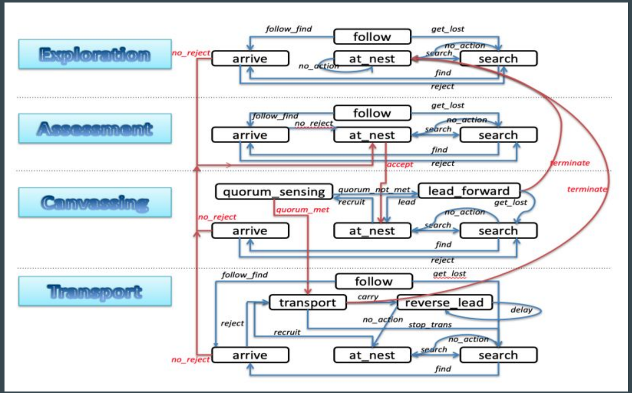
- Model captures biological measurements from empirical studies (Zhao, Lynch, Pratt, 2020)
  - Bio-plausible
  - Predicts less studied behaviors of ants
- **Challenge:** Lack theoretical bounds on the running time of the algorithm



Phase transitions of an ant

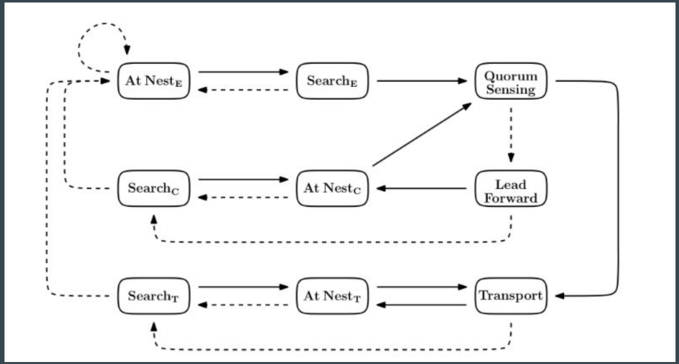
Simplifications

Before




```python
self.state_name = _state_name # name of the state_name
self.home_nest = _home_nest # default to 0
self.candidate_nest = _candidate_nest
self.transitioned = _transitioned
self.location = _home_nest
self.phase = "E" # how much ant is committed to the candidate nest
self.old_candidate_nest = -1 # only used for reject action
self.terminate = 0
```

After

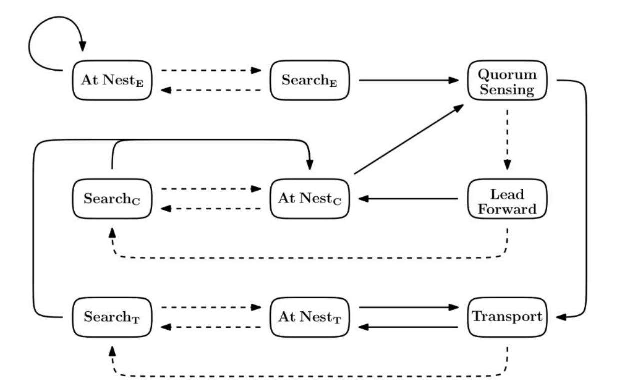


State,
Location

Adjustable Parameters

Parameter	Value	Source
quality coefficient μ_q	0.25	trial-and-error from [13]
population coefficient μ_p	0.35	trial-and-error from [13]
quorum threshold θ	0.15	[9, 2]
search constant c_s	0.025	trial-and-error from [13]
follow constant c_f	0.4	[4, 8]
lead forward constant c_ℓ	0.6	trial-and-error from [13]
transport constant c_t	0.7	[10]
λ	8	trial-and-error from [13]

State Transitions



$$\Pr[\mathbf{u} = \text{advance} \mid a.state = \text{At Nest}_i] = \left(1 + e^{-\lambda\left(\mu_q \cdot q + \mu_p \cdot \frac{p}{n}\right)}\right)^{-1} \quad \text{for } i \in \{\text{E, C, T}\}$$

$$\Pr[\mathbf{u} = \text{advance} \mid a.state = \text{Search}_i] = c_s \cdot \left(1 + e^{-\lambda\left(\mu_q \cdot (q' - q) + \mu_p \cdot \frac{p' - p}{n}\right)}\right)^{-1} \quad \text{for } i \in \{\text{E, C, T}\}$$

$$\Pr[\mathbf{u} = \text{advance} \mid a.state = \text{Quorum Sensing}] = \begin{cases} 1 & \text{if quorum has been met — that is, } p_a > \theta \cdot n_a \\ & \text{and } a.location \text{ has not dropped out of competition} \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr[\mathbf{u} = \text{advance} \mid a.state = \text{Transport}] = c_t$$

$$\Pr[\mathbf{u} = \text{advance} \mid a.state = \text{Lead Forward}] = \begin{cases} c_\ell & \text{if } q > q' \\ 0 & \text{otherwise} \end{cases}$$

House-Hunting Algorithm

Algorithm 1: One Round of the HOUSEHUNTING Algorithm

M: a set of ants, initially $\emptyset$

for $i = 1$ *to* n_a **do**

  if $a_{P(i)} \notin M$ **then**

    $action, n' :=$ **select_action**$(\mathbf{a_{P(i)}})$

    $a' :=$ **select_ant**$(\mathbf{a_{P(i)}}, \mathbf{n'}, \mathbf{action})$

    if $a' \in M$ **then**

      $a' \leftarrow$ *null*

    transition$(\mathbf{a_{P(i)}}, \mathbf{a'}, \mathbf{n'}, \mathbf{action})$

    $M := M \cup \{a_{P(i)}\} \cup \{a'\}$

A Lower Bound on Number of Rounds Required

Proof inspired by Ghaffari, Musco, Radeva, Lynch, 2015.

Method:

- Lower bound the probability that a constant fraction of the ants goes to the new nest on any given round.
- Chernoff Bound

Result:

- An algorithm with n ants requires **$\Omega(\log n)$** rounds to converge with high probability.

An Expected Upper Bound for Single Nest Emigrations

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Theorem 4.4. *If the quorum threshold satisfies* $1 - \frac{a(\epsilon)}{n_a} < \theta < \frac{a(\epsilon)}{n_a}$, *then* $\mathbb{E}[R_\epsilon] = O(\log n)$.

$n_a =$ the number of active ants

$$a(\epsilon) \approx n_a \left(\frac{1-\epsilon}{1+e^{-\lambda(\mu_q(q_1-q_0)-\mu_p)}} \right)$$

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Lower bound: transports to inferior nest taper off
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$\lambda = 8,\ \mu_q = .25,\ \mu_p = .35,\ q_1 = 3,\ q_0 = 0,\ \epsilon = .00001$
$\theta \in (.0392, .9608)$ (reasonable bounds)
$\theta \approx .15$

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Future Work:

1. Generalize this result to environments with multiple nests
2. $$1 - \frac{a(\epsilon)}{n_a} < \frac{a(\epsilon)}{n_a} \implies q_1 - q_0 > \frac{\mu_p}{\mu_q}$$

   Test this prediction about the relative qualities of nests in real experiments with ants

Implications

- Gain a better understanding of the biological behavior of ants.
- Studying biologically-inspired algorithms can help engineer better distributed computer systems.
    - Robot swarms

Thank you.

Questions?